

Province of the
EASTERN CAPE
EDUCATION

MATHEMATICS P1

MEMORANDUM

COMMON TEST

JUNE 2014

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 125

TIME : 2½ hours

This memorandum consists of 8 pages.

1.1.1	$(x+6)(x-1)=0$ $x=-6$ or $x=1$	✓ factors ✓✓ Answers	(3)
1.1.2	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $= \frac{-(4) \pm \sqrt{(4)^2 - 4(-3)(2)}}{2(-3)}$ $= -0,39$ or $1,72$	✓ substitution into quadratic formula ✓✓ answers	(3)
1.1.3	P To		(3)
1.1.4	$2^x(2^3 - 3 \cdot 2^{-1}) = 104$ $2^x \left(8 - \frac{3}{2}\right) = 104$ $2^x = 16$ $2^x = 2^4$ $\therefore x = 4$	✓ removal of common factor ✓ simplifying bracket ✓ 16 ✓ writing 16 as base 2 ✓ answer	(5)
1.2	$\sqrt{36 \times 2x} - \sqrt{49 \times 2x} + 2\sqrt{144 \times 2x}$ $6\sqrt{2x} - 7\sqrt{2x} + 24\sqrt{2x}$ $= 23\sqrt{2x}$	✓ perfect squares ✓ for writing all as mixed surds ✓ answers	(3)
1.3	$y = 2x - 2$ $2x - 2 = (x-2)(x-1)$ $2x - 2 = x^2 - 3x + 2$ $x^2 - 5x + 4 = 0$ $(x-1)(x-4) = 0$ $x = 1$ or $x = 4$ $y = 0$ or $y = 6$ OR $x - 1 = \frac{y}{2}$ $y = \left(\frac{y}{2} - 1\right)\left(\frac{y}{2}\right)$ $y = \frac{y^2}{4} - \frac{y}{2}$ $4y = y^2 - 2y$ $y^2 - 6y = 0$ $y(y-6) = 0$ $y = 0$ or $y = 6$ $x = 1$ or $x = 4$	✓ for y as subject ✓ substitution of y ✓ form of equation ✓ factors ✓ x values ✓ y values ✓ for y as subject ✓ substitution of y ✓ form of equation ✓ factors ✓ y values ✓ x values	(6)
			[23]

1.1.3.

$$\frac{x^2}{x+2} \leq 0 \quad -$$

poi :

$$x^2 = 0$$

$$x = 0$$

$$x+2 = 0$$

$$x = -2$$

$$\begin{array}{ccccccc} \ominus & \text{UD} & & & & & \\ & | & + & 0 & + & & \\ \hline & -2 & & 0 & & & \end{array}$$

$$x < -2$$



2.1.1	11 ; 14	✓ answer (both terms)	(1)	
2.1.2	$T_n = 3n - 1$ Now if $3n - 1 = n^2$ $n^2 - 3n + 1 = 0$ $n = \frac{3 \pm \sqrt{5}}{2}$ $\Rightarrow n$ is irrational But n must be natural \therefore No term is a perfect square ($b^2 - 4ac \neq$ a perfect square)	or $b^2 - 4ac = (-3)^2 - 4(1)(1)$ $= 5$ $\Delta \neq$ a perfect square	✓✓ for nth term ✓ for equating nth term to n^2 ✓ for value of n ✓ for concluding n is irrational and for deducing n is natural	(5)
2.2.1	$T_n = 2n - 1$ for the first difference sequence $T_6 = 2(6) - 1 = 11$ $\therefore T_6$ of original seq. = $35 - 11 = 24$ $T_5 = 2(5) - 1 = 9$ $\therefore T_5$ of original seq. = $24 - 9 = 15$	✓ 6 th term of 1 st difference seq. ✓ 6 th term of quadratic seq. ✓ 5 th term of 1 st difference seq. ✓ 5 th term of quadratic sequence	(4)	
2.2.2	$36a + 6b + c = 24 \rightarrow (1)$ $25a + 5b + c = 15 \rightarrow (2)$ $49a + 7b + c = 35 \rightarrow (3)$ (1) - (2): $11a + b = 9 \rightarrow (4)$ (3) - (1) $13a + b = 11 \rightarrow (5)$ (5) - (4) $2a = 2$ $\therefore a = 1$ $b = -2$ $T_n = n^2 - 2n$ OR $2a = 2$ $\therefore a = 1$ $c = 0$ $T_n = an^2 + bn + c$ $35 = 1(7)^2 + b(7) + 0$ $-14 = 7b$ $b = -2$ $T_n = n^2 - 2n$	✓ $11a + b = 9$ ✓ $13a + b = 11$ ✓ a value ✓ b value ✓ answer ✓ a value ✓ c value ✓ $T_7 = 35$ ✓ b value ✓ answer	(5)	

<p>2.3</p>	$S_n = a + ar + ar^2 + \dots + ar^{n-1} \rightarrow (1)$ $rS_n = ar + ar^2 + \dots + ar^n \rightarrow (2)$ <p>(2) - (1):</p> $rS_n - S_n = ar^n - a$ $S_n(r - 1) = a(r^n - 1)$ $\therefore S_n = \frac{a(r^n - 1)}{r - 1}$	<ul style="list-style-type: none"> ✓ for equation (1) ✓ for equation (2) ✓ subtraction on LHS and RHS ✓ factorising 	<p>(4)</p>
<p>2.4</p>	$S_n = \frac{a(r^n - 1)}{r - 1}$ $531440 = \frac{2(3^n - 1)}{3 - 1}$ $3^n = 531441$ $n = \log_3 531441$ $n = 12$	<ul style="list-style-type: none"> ✓✓ substitution ✓ for 3^n as subject ✓ writing in log form ✓ answer 	<p>(5)</p>
			<p>[24]</p>

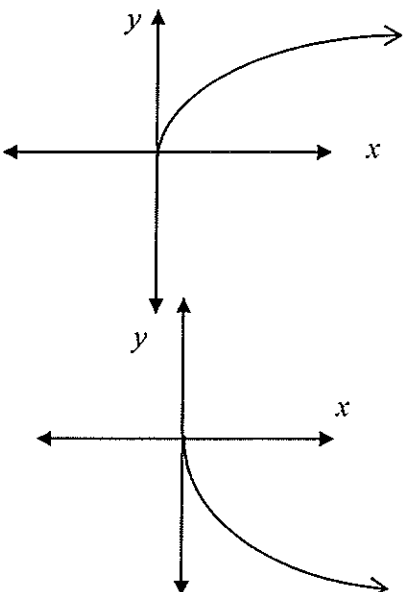
QUESTION 3

<p>3.1</p>		<p>f:</p> <ul style="list-style-type: none"> ✓ turning point ✓ y - intercept ✓✓ x - intercepts: 1 +ve and 1 -ve ✓ shape <p>g:</p> <ul style="list-style-type: none"> ✓ Horizontal asymptote ✓ y - intercept ✓ shape 	<p>(8)</p>
<p>3.2.1</p>	<p>$t = 6$</p>	<p>✓✓ answer</p>	<p>(2)</p>
<p>3.2.2</p>	<p>$t = 2$</p>	<p>✓✓ answer</p>	<p>(2)</p>
<p>3.3</p>	<p>$y = 2$</p>	<p>✓✓ answer</p>	<p>(2)</p>
			<p>[14]</p>

QUESTION 4

4.1	$p = -3$ $q = 4$ $y = \frac{a}{x-3} + 4$ Now subst. $A(4;6)$: $6 = \frac{a}{4-3} + 4$ $\therefore a = 2$	✓ p value ✓ q value ✓ subst. p, q and point A ✓ a value	(4)
4.2	$y \in R; y \neq 2$	✓✓ answer	(2)
4.3	$4 = 3 + c$ $\therefore c = 1$ OR $y = (x + p) + q$ $y = (x - 3) + 4$ $y = x + 1$ $\therefore c = 1$	✓✓ subst. of point A ✓ answer ✓ substitution of p and q values ✓ equation of line of symmetry ✓ answer	(3)
			[9]

QUESTION 5

5.1	$y = 5^x$	✓✓ answer	(2)
5.2.1	$y = \pm\sqrt{x}$	✓✓ answer	(2)
5.2.2	h is a many-to-one-function OR For each x -value, there is more than 1 y -value OR If you draw a vertical line parallel to the y -axis, it will cut h twice.	✓✓ answer ✓✓ answer ✓✓ answer	(2)
5.2.3	$x \leq 0$ $x \geq 0$	✓ answer ✓ answer	(2)
5.2.4		✓ shape ✓ intercept ✓ shape ✓ intercept	(4)

5.2.5	$\sqrt{x} \leq 2$ So consider $\sqrt{x} = 2$ $x = 4$ $\therefore 0 \leq x \leq 4$	✓✓ answer[end points + inequality]	(2)
			[14]

QUESTION 6

6.1	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^3 - (x)^3}{h}$ $= \lim_{h \rightarrow 0} \frac{(x^3 + 3x^2h + 3xh^2 + h^3) - x^3}{h}$ $= \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h}$ $= 3x^2$ <p>OR</p> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h)^3 - (x)^3}{h}$ $= \lim_{h \rightarrow 0} \frac{(x+h-x)(x^2 + 2xh + h^2 + xh + x^2 + x^2)}{h}$ $= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2)$ $= 3x^2$	✓ formula ✓ substitution ✓ simplifying ✓ factorising ✓ answer ✓ formula ✓ substitution ✓ factorising ✓ simplifying ✓ answer	(5)
6.2.1	$y = x^2 - x$ $\frac{dy}{dx} = 2x - 1$	✓✓ for simplifying ✓✓ both answers	(4)
6.2.2	$h(x) = x^{\frac{1}{6}} - 3x^{\frac{1}{2}}$ $h'(x) = \frac{1}{6}x^{-\frac{5}{6}} - \frac{3}{2}x^{-\frac{1}{2}}$	✓✓ rewriting both terms in exponential form ✓✓ both answers	(4)
			[13]

QUESTION 7

7.1	A(2; 0)	✓✓ for x and y coordinates	(2)
7.2	$y = a(x - x_1)(x - x_2)(x - x_3)$ $-2 = a(0+1)(0+1)(0-2)$ $-2 = -2a$ $\therefore a = 1$ $y = (x+1)(x+1)(x-2)$ $= (x^2 + 2x + 1)(x-2)$ $= x^3 - 3x - 2$ $\therefore c = 3$ OR $f'(x) = 3ax^2 - c$ $f'(-1) = 3a(-1)^2 - c = 0$ $3a - c = 0 \rightarrow (1)$ <i>Also</i> $f(-1) = a(-1)^3 - c(-1) - 2$ $-a + c - 2 = 0$ $-a + c = 2 \rightarrow (2)$ $(1) + (2):$ $2a = 2$ $\therefore a = 1$ <i>subst.in(2):</i> $c = 3$ OR $f(x) = a(x+1)^2(x-2)$ $= a(x-2)(x^2 + 2x + 1)$ $= a(x^3 + 2x^2 + x - 2x^2 - 4x - 2)$ $= a(x^3 - 3x - 2)$ $\therefore a = 1$ and $c = 3$	✓✓ substituting x intercepts and y intercept into formula ✓ for $-2 = -2a$ ✓ simplifying any two brackets ✓ cubic function ✓ for derivative ✓ subst. $x = -1$ into derivative ✓ for subst. $x = -1$ into original equation f ✓ for $2a = 2$ ✓ c value ✓ for subst. x -intercepts into equation ✓ for squaring binomial ✓ for expanding ✓ for collecting like terms ✓ for answers	(5)
7.3	$\frac{dy}{dx} = 3x^2 - 3$ $3x^2 - 3 = 0$ $3(x+1)(x-1) = 0$ $\therefore x = -1$ or $x = 1$ $\Rightarrow y = 0$ or $y = -4$ B(1; -4)	<div style="border: 1px solid black; padding: 5px; display: inline-block;"> Also consider second derivative method </div> ✓ derivative and equal to 0 ✓ x - values ✓ y - values ✓ coordinates of B	(4)
7.4	$x = 0$	✓✓ answer	(2)
7.5	$k > 0$ or $k < -4$	✓✓✓ answers and accuracy mark for the word OR	(3)
7.6	$-1 < x < 1$	✓✓ end points and inequality	(2)
			[18]

QUESTION 8

8.1	$V = l \times b \times h$ $l = 2x \times x \times h$ $h = \frac{1}{2x^2}$	✓ substituting into volume formula ✓ h value in terms of x	(2)
8.2	$C = 2x^2 \times R200 + 6xh \times R120$ $= 400x^2 + 720x \left(\frac{1}{2x^2} \right)$ $= 400x^2 + 360x^{-1}$	✓ cost of base ✓ cost of sides ✓ substitution of h value	(3)
8.3	$C'(x) = 800x - 360x^{-2}$ $800x = \frac{360}{x^2}$ $x^3 = \frac{360}{800}$ $x = 0,77 \text{ m}$ <p><i>Minimum Cost</i></p> $= 400(0,77)^2 + 360(0,77)^{-1}$ $= R704,69 \text{ or } R704,68$	✓ derivative = 0 ✓ for making x^3 the subject ✓ x value ✓ substitution ✓ answer	(5)
			[10]

TOTAL MARKS: 125